Analysis of Intensity and Phase Noise of Solitary Semiconductor Lasers Operating in Single-Mode

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Abstract- Semiconductor lasers are being used on large scale now-a-days in many areas of optical data storage and optical communication system. Although laser diodes offer cost effective and reliable means of generating coherent light, semiconductor lasers often involve various noise and instability problems due to fluctuation of photon, its phase and carrier numbers. In this paper, we discuss the intensity and phase noise of solitary semiconductor lasers operating in single-mode. To overcome limitations of the small-signal analysis we use direct numerical integration of the self-consistent rate equations for photon number, its phase and carrier density. Langevin noise sources for photon number and its phase have been introduced to the rate equations to include fluctuations due to spontaneous emission and the process of carrier recombination. For the purpose of investigation, the laser rate equations are applied to 850 nm GaAs lasers. Fast Fourier transform (FFT) has been used to calculate the frequency spectra of both intensity and phase noise. Transient behavior of semiconductor laser is also described that is significant in determining the noise characteristics of the laser output. Noise characteristics have been demonstrated for different injection currents and described in terms of the photon and carrier population fluctuations. Results show that intensity and phase noise decrease with the increase of the injection current density and linewidths were decreasing substantially with the increasing injection current as well.

Key Words: Laser rate equations; quantum noise; intensity noise; phase noise; linewidths; noise spectrum; Langevin noise source.

1. INTRODUCTION

Semiconductor lasers are one of the most essential optoelectronic devices in modern age. Depending on gain media, they have a complex multi-level design with accurate preciseness. The optical gain is usually achieved by stimulated emission at an inter-band transition under conditions of a high carrier density in the conduction band [1]. Since the invention and advancement of the first laser diode in four decades, impressive progress has been accomplished so far. The term single-mode operation in lasers, being ambiguous, indicates either single-transverse mode operation or single-frequency operation. Because single-frequency operation is not sufficient to introduce spatially varying loss or gain, it is very difficult to attain single-frequency operation than single-transverse-mode operation [2]. Since semiconductor laser offers cost effective and reliable means of generating coherent light suitable for various applications, they are in widespread use today in many areas from CD, DVD and other forms of data storage through to communication links. But the laser diode performance is largely affected by various noise and instability problems. Early noise calculations were based on small signal analysis developed by McCumber [1] and Haug applied that to semiconductor lasers [3]. Linearization of the rate equations following the small signal approximation brings about the analytical treatment which was applied in most of the previous calculations [4]-[11]. However, information concerning the instantaneous fluctuations of the photon and carrier numbers and also frequency was missed in such small signal calculations. To overcome the limitations of the small signal analysis direct numerical integration of the rate equations has been exercised [12]-[22].

Noise problem is accompanied with laser since its invention. The term “noise” in semiconductor laser refers to random fluctuations of various output parameters due to quantum-mechanical effects along with many internal and external influences which limits the performance of the laser. Two variance of quantum noise- intensity noise and phase noise correspond to intrinsic fluctuations in the photon number, carrier number and phase generated during the quantum interaction processes of the lasing field with the injected charge carriers [23], [24]. So, it is imperative to analyze various types of laser noise in order to achieve the paramount performance of the semiconductor laser.

In this paper, we have used the rate equations of the photon number $S(t)$ and injected electron number $N(t)$ as well as the phase $\theta(t)$ with a view to considering the noise characteristics of semiconductor lasers. Theoretical analysis has been made by solving of the laser rate equations by numerical simulation including Langevin noise sources to include fluctuated spontaneous emission [25]. In order to generate frequency fluctuation, both the carrier number fluctuations and the random process of spontaneous emission need to be acknowledged. Considering the existence of only the fundamental transverse mode, we have applied the rate equation model to 850 nm GaAs laser and observed the frequency spectra of intensity and phase noise following numerical simulation.

2. THE RATE EQUATION MODEL

The rate equations of semiconductor lasers can be presented as follows. [26]
For photon phase:
\[
\frac{d\theta}{dt} = \frac{\alpha a \xi}{2V} (N - \bar{N}) + F_p(t)
\]  
(2)

For injected carrier (electron) numbers:
\[
\frac{dN}{dt} = -AS + \frac{N}{\tau_e} + \frac{I}{e}
\]  
(3)

In eqn. (1) \( G \) is the gain of single-mode laser with wavelength \( \lambda \), given by-
\[
G = A \cdot B \cdot S
\]  
(4)
\[
G_{th} = \frac{c}{\hbar} k \left( \frac{1}{2L R_R R_p} \right)
\]  
(5)

\( A \) is the linear gain which can be expressed as,
\[
A = \frac{a \xi}{V} (N - N_g)
\]  
(6)

\( B \) is the self-suppression coefficient written as,
\[
B = \frac{9}{4} \frac{\hbar \omega}{e \tau_m} a \xi V (N - N_g)
\]  
(7)

Other parameters are: \( \omega \)-differential gain coefficient, \( \xi \)-field confinement factor, \( V \)-volume of the active region, \( L \)-length of the active region, \( \alpha \)-linewidth enhancement factor, \( \bar{N} \)-time average value of \( N(t) \), \( \tau_e \)-electron lifetime, \( I \)-injection current, \( e \)-electron charge, \( k \)-internal loss of the laser cavity, \( N_e \)-electron number at transparency, \( \hbar \)-reduced Planck constant, \( \tau_m \)-intradab relaxation time, \( R_0 \)-dipole moment, \( N_g \)-electron number characterizing self-suppression coefficient.

The functions \( F_p(t) \) and \( F_e(t) \) are Langevin noise sources for photon numbers and its phase, respectively. These noise sources are given by,
\[
F_p(t) = \frac{V_{SS}}{\Delta t} g_p
\]
(8)
\[
F_e(t) = \frac{V_{SS}}{\Delta t} g_e
\]
(9)

where \( V_{SS} \) is the variance of autocorrelation given by,
\[
V_{SS} = \frac{a \xi}{V} (N + N_g) + G_{th}\left( \frac{a \xi V}{N} \right) + \frac{a \xi V}{V}
\]  
(10)

\( g_p \) and \( g_e \) are the Gaussian random numbers in ranges of -1 < \( g_p < +1 \) and -1 < \( g_e < +1 \), and \( \Delta t \) time-step of the calculation.

Box-Muller Transformation method [27] has been used so that these Gaussian random variables can be generated from naturally distributed random numbers. To generate these two uniformly distributed random variables \( u_1 \) and \( u_2 \) have been used in the range between -1 to +1. The following equations are used to obtain \( g_p \) and \( g_e \).
\[
g_p = \frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{u_2} \frac{1}{\sqrt{2 \pi}} \cos(2 \pi u_1) du_1 \cdot \cdot \cdot a = s, \theta
\]  
(11)

The output power \( P(t) \) from the front facet of semiconductor lasers is given by [25],
\[
P(t) = \frac{\hbar \omega}{2 \pi \lambda} \ln \left( |\sqrt{R_R R_f}||1 - R_p| \right) S(t)
\]  
(12)

where, \( c \) is the speed of light in vacuum, \( n_r \) is the refractive index of the active region, \( h \) is the photon energy of the emitted light, \( R_f \) is the power reflectivity of the front facet and \( R_b \) is the power reflectivity of the back facet.

In the previous calculations used by McCumber [1] and Haug [3], the time-fluctuating components are transformed into Fourier frequency components from which the relative intensity noise, phase noise and linewidth are calculated. But in our numerical approach in this report RIN (relative intensity noise), FN (frequency or phase noise) and linewidth can be calculated from the optical power fluctuations of semiconductor laser.
\[
\delta P(t) = \overline{P(t)} - \overline{P}
\]  
(13)

where, \( \overline{P} \) = time average optical power.

The power fluctuation can be obtained from the rate equations of photon number and carrier number by integrating them numerically and calculating the spectrum of intensity fluctuations.

The RIN and FN spectrum can be originally defined as the Fourier transform of the autocorrelation function written as,
\[
RIN = \frac{1}{\overline{P}^2} \left| FFT[\delta P(t)] \right|
\]  
(14)
\[
FN = \frac{1}{\overline{F}^2} \left| FFT[\Delta \nu(t)] \right|
\]  
(15)

where, variation of optical phase is given as-
\[
\Delta \nu(t) = \frac{1}{2 \pi} \frac{d \theta}{dt} = \frac{1}{\Delta t} \frac{\Delta \theta(t)}{\Delta t}
\]  
(16)

Laser linewidth is the full-width at half-maximum (FWHM) spectrum of the laser operating in single-mode. It is determined from the low frequency component of the frequency or phase noise as
\[
\text{Laser Linewidth} = \Delta f = 4 \pi FN_{\nu=0}
\]  
(17)

3. NUMERICAL SIMULATION

Our primary purpose is to attain the photon number \( S(t) \), phase number \( \theta(t) \) and carrier number \( N(t) \) with comparable noise terms by numerical simulation. The standard parameter values for GaAs laser needed for simulation are- \( \alpha = 2.75 \times 10^{-12} \text{ m}^2 \text{ s}^{-1} \), \( |R_0|^2 = 2.8 \times 10^{-17} \text{ C}^2 \text{ m}^2 \), \( \xi = 0.2 \), \( V = 100 \mu \text{m}^3 \), \( L = 300 \mu \text{m} \), \( d = 0.11 \mu \text{m} \), \( \tau_m = 0.1 \text{ ns} \), \( \tau_e = 2.79 \text{ ns} \), \( \lambda = 850 \text{ nm} \), \( k = 10 \text{ cm}^{-1} \), \( N_g = 1.7 \times 10^{10} \), \( N_s = 1.7 \times 10^9 \), \( n_r = 3.6 \), \( R_f = 0.3 \) and \( R_b = 0.6 \).

The fourth-order Runge-Kutta method has been used with a view to solving the rate equations. A short time interval of \( \Delta t = 5 \text{ ps} \) has been used for executing the numerical integrations. The purposes of taking such a small value of \( \Delta t \) for calculation are to generate noise sources approximately characterizing a white noise spectrum up to a frequency of 200 GHz and to examine the behavior of the laser both before and after relaxation frequency. We have expanded the integration to a time period as long as 10 \( \mu \text{ s} \), requiring more than 1 million integration steps, so that we can find out the intensity noise as low as 100 kHz.

We have generated Gaussian random variable \( g \) using the Box-Muller transformation method [27] where two uniformly distributed random variables \( u_1 \) and \( u_2 \) are first taken. These variables are generated within the range -1 and +1. By using the Box-Muller transformation method, the Gaussian random
variables for photon number and phase are obtained from $u_1$ and $u_2$ using eqn. (11).

4. RESULT AND DISCUSSION

The time varying profiles of the photon number $S(t)$, the carrier number $N(t)$ and frequency fluctuation $\Delta v(t)$ can be realized and plotted through numerical simulation. For an injection current $I$ equal to 1.7 times the threshold value $I_{th}$, these profiles are shown in Figures 1, 2 and 3, respectively.

In Fig. 1, it can be seen that during steady state operation photon number continues to fluctuate close to its average value $\bar{S}$. This happens because Langevin noise sources drive the rate equations so that these physical quantities fluctuate around their dc values.

The same phenomena can be observed for the time varying profile of the carrier (electron) number as well as the phase as shown in Figures 2 and 3, respectively.

It is spontaneous emission that contributes to a comparably minimal amount of output light whereas stimulated emission produces the main output in semiconductor lasers like any other kind of lasers. Both the interaction between the photon population in the cavity and the fraction of injected carriers, excessive of the equilibrium threshold value accelerate the way of response of the output intensity light to the injection current. [30] It can be seen from Figures 1 and 2 how both the photon and injected carrier population response at steady state.

Both the RIN spectrum and the FN spectrum can be originally defined as the Fourier transform of the autocorrelation function. In this paper, RIN and FN spectrum has been obtained using the fast Fourier transform or FFT.

![Fig. 1: Time varying profile of photon number.](image1)

![Fig. 2: Time varying profile of electron number.](image2)

![Fig. 3: Time varying profile of frequency fluctuation.](image3)

![Fig. 4: (a) RIN Spectrum for different values of injection current, (b) FN Spectrum for different values of injection current.](image4)

Relative Intensity Noise: $\text{RIN} = \frac{1}{\mathcal{P}^2} \left| \text{FFT}(\delta P(t)) \right|^2$

where, $\mathcal{P}$ = time average optical power,
\[ \Delta t = \text{time-step of the calculation}, \]
\[ T = \text{total time period of the calculation and} \]
\[ \delta P(t) = P(t) - \bar{P} = \text{optical power fluctuation}. \]

**Frequency or Phase Noise:** 
\[ FN = \frac{\Delta t^2}{T} \left| \text{FFT} \left[ \Delta v(t) \right] \right|^2 \]
\[ \Delta v(t) = \frac{1}{2\pi} \frac{d\theta}{dt} = \frac{1}{2\pi} \frac{\Delta \theta(t)}{\Delta t} = \text{variation in optical phase.} \]

A great deal of noise fluctuations, greater than the natural quantum noise of the photon stream, are present in the output of all types of lasers due to the fact that the quantum fluctuations in the electron and photon population are amplified in the optical resonator. The discrete and random nature of the emission and recombination processes may also be affected by this. The effect of fluctuations in the photon and carrier populations on the output of the laser is similar to that in which would be produced by deliberate modulation of the two populations. While analyzing the overall effect, complementary processes of photon emission or absorption and electron recombination or generation, being correlated in time, must be taken into consideration. The pumping current accelerates the response of the laser with respect to the fluctuations. The noise is magnified over a certain band of frequencies around the relaxation frequency by a resonant interaction.

A comparative study has been made on the effect of injection current on RIN spectrum and FN spectrum by applying three different values of injection current - \( I = 1.1I_\text{th}, \) \( I = 1.5I_\text{th}, \) and \( I = 2.0I_\text{th}. \) The corresponding simulated spectra of RIN and FN are shown in Figs. 4(a) and 4(b), respectively. Observable peaks are obtained at injection currents \( I \) greater than threshold current. This may be attributed to the fact that for injection current \( I \) greater than \( I_\text{th}, \) the noise is caused by photon fluctuation and for injection current \( I \) smaller than \( I_\text{th}, \) the noise is caused by electron fluctuation.

Figs. 4(a) and 4(b) show that, both intensity and phase noise were decreased with the increase of injection current density \( I. \) Figures also show pronounced peaks for both RIN and FN around the relaxation oscillation frequencies that were experimentally detected in [31]. At the same time, increasing the injection current results in shifting of the relaxation oscillation peaks towards higher frequencies. This is due to the fact that the repetition of fluctuations of both \( \delta P(t) \) and \( \Delta v(t) \) becomes faster with increasing \( I. \) On the other hand, the suppression of the fluctuations leads to a decrease in the level of RIN and FN.

Laser linewidth can be calculated from eqn. (17) applying the condition of FN to very low frequencies. Although this is very difficult when using the short integration step \( \Delta t = 5 \) ps from the computational point of view, the flatness of the curve at the low-frequency side enabled us to approximately calculate \( \Delta f \) at frequencies as low as 100 MHz. [25]

These show the rapid narrowing of \( \Delta f \) with increasing \( I \) which have been plotted in Fig. 5. Since there is no stimulated emission below threshold current, laser linewidth cannot be determined for those values. The decrease of \( \Delta f \) with \( I \) matches the corresponding decrease of the FN reported in Fig. 4(b). If we consider the linewidth for a particular injection level, say, \( I = 1.5I_\text{th}, \) the calculated value we get is 10.91 MHz which is comparable to the value 11.0 MHz obtained from Schawlow-Townes relation [23], based on the small-signal approximation. Therefore, we can come to an agreement that the results satisfy the modified Schawlow-Townes relation that describes the dependence of the linewidth \( \Delta f \) on the injection current \( I. \)

When the laser current is increased from the threshold, the relative output noise below 100 MHz or so shows a maximum around threshold. But later this noise decrease with a further increase in current. The resonant enhancement also effect the noise centered at frequencies higher than a few hundred megahertz. The centre frequency in this case increases with current and moves into the gigahertz region for currents greater than \( I = 1.1I_\text{th}. \) Therefore, decrease in frequency fluctuation is achieved with increasing current \( I \) that is evident in the frequency fluctuation vs time graph in Fig. 6.

Laser output power \( P(t) \) varies with different injection currents \( I \) as shown in Fig. 7. The plotted fluctuations are far from the relaxation regime. The plot features that the repetition of the fluctuation becomes faster with increasing current \( I \), which indicates that the relaxation frequency has increased with injection current \( I \) as is evident in Fig. 4(a).
5. CONCLUSION

The noise characteristics of semiconductor lasers have been studied in this paper in terms of photon number and its phase fluctuations and the time varying carrier populations as well. We have analyzed the intensity and phase noise of semiconductor laser as well as laser linewidth through numerical simulation of the laser rate equations including Langevin noise sources for the photon number and its phase that account for the generation of the fluctuations. The noise characteristics have been demonstrated under different injection current values and found to be decreasing with the increase of injection current density. Laser linewidths were also decreasing with the increase of carrier injection. The intensity noise present in the output of laser diodes limit their reliability when applied as light sources in optical communication systems, optical discs, etc. Analysis of the laser type noise thus becomes crucial in improving the efficiency of semiconductor lasers.

REFERENCES

