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The Mossbauer Rotor Effect- Relativistic Electrodynamics in Uniformly Rotating Frames

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Abstract- In the current paper we present the exact formalism of the Mossbauer effect as an application of the relativistic electrodynamics in rotating frame. We show that the results are in perfect agreement with the predictions of the special theory of relativity as applied to rotating frames. The results show that the experiment is a pure expression of the transverse Doppler effect. In the process of deriving the formalism, we have cleared a few misconceptions about the expected numerical results and we exposed a potential source of very small systematic errors.

Key Words: Mossbauer effect, Mossbauer rotor experiment, Uniform rotation motion, Planar electromagnetic waves, Relativistic Doppler effect, Transverse Doppler effect, Relativistic aberration, Relativistic electrodynamics in uniformly rotating frames.

1. INTRODUCTION – THE MOSSBAUER ROTOR EXPERIMENT

A confirmation of the relativistic Doppler effect was achieved by the Mössbauer rotor experiment [1]. Gamma rays are sent from a source in the middle of a rotating disk (see Fig.1) to an absorber at the rim and a stationary counter is placed beyond the absorber. The characteristic resonance absorption frequency of the moving absorber at the rim should decrease due to time dilation, so the transmission of gamma rays through the absorber increases, which is subsequently measured by the stationary counter beyond the absorber. The maximal deviation from time dilation was 10^{-5}. Such experiments were performed by Hay et al. [2,3] , Champeney et al.[4,6] and by Kündig [5].

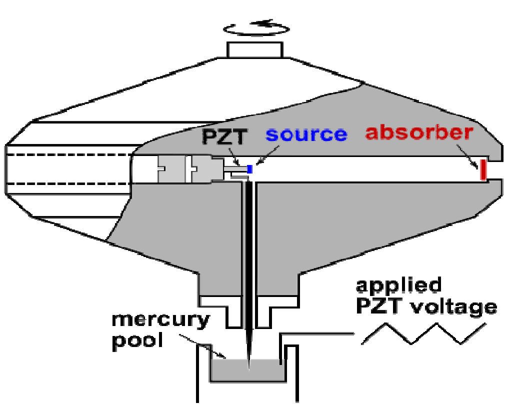

Fig. 1: The Mossbauer rotor experiment

2. THE MOSSBAUER EXPERIMENT THEORY EXPLAINED IN ROTATING FRAMES

In this section we apply the formalism derived in [8,9] in order to obtain the explanation of the Mossbauer experiment Assume that a planar wave is propagating along the y' axis in the accelerated frame $S'(\tau)$. The wave has the electric component $\mathbf{E}_x^{'}$ and the magnetic component $\mathbf{B}_z^{'}$ along the x' and z' axes, respectively. The components equations are (see Fig.2):

$$\begin{aligned}\mathbf{E}_x^{'} &= E_{0x}^{'}\cos(\Omega' t' - k_y^{'} y' + \theta')\mathbf{e}_x \\ \mathbf{B}_z^{'} &= B_{0z}^{'}\cos(\Omega' t' - k_y^{'} y' + \theta')\mathbf{e}_z\end{aligned} \quad (1)$$

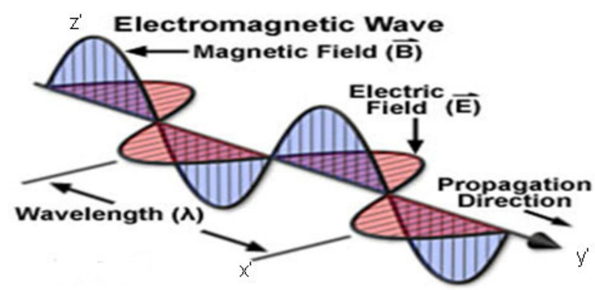

Fig. 2: The electromagnetic wave

On the other hand, from [8,9]:

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{bmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \quad (2)$$

In frame S, the wave equation is [8,9] either:

$$\begin{aligned}
&E_x = (\gamma\cos\alpha\cos\beta + \sin\alpha\sin\beta)E_x^{'} - (u\gamma\cos\beta)B_z^{'}\} = \\
&= [(\gamma\cos\alpha\cos\beta + \sin\alpha\sin\beta)E_{0x}^{'} - (u\gamma\cos\beta)B_{0z}^{'}]* \\
&*\cos[(\Omega' b_{44} - k_y^{'} b_{24})t - (k_y^{'} b_{22} - \Omega' b_{42})y - (k_y^{'} b_{21} - \Omega' b_{41})x + \theta'] \\
&E_y = [(\gamma\cos\alpha\sin\beta - \sin\alpha\cos\beta)E_{0x}^{'} - (u\gamma\sin\beta)B_{0z}^{'}]* \\
&*\cos[(\Omega' b_{44} - k_y^{'} b_{24})t - (k_y^{'} b_{22} - \Omega' b_{42})y - (k_y^{'} b_{21} - \Omega' b_{41})x + \theta'] \\
&B_z = \gamma(B_{0z}^{'} - \frac{u\cos\alpha}{c^2}E_{0x}^{'})* \\
&*\cos[(\Omega' b_{44} - k_y^{'} b_{24})t - (k_y^{'} b_{22} - \Omega' b_{42})y - (k_y^{'} b_{21} - \Omega' b_{41})x + \theta']
\end{aligned} \quad (3)$$

or

$$\mathbf{E}_x = E_{0x}\cos(\Omega t - k_x x - k_y y - k_z z + \theta)\mathbf{e}_x$$
$$\mathbf{E}_y = E_{0y}\cos(\Omega t - k_x x - k_y y - k_z z + \theta)\mathbf{e}_y \quad (4)$$
$$\mathbf{B}_z = B_{0z}\cos(\Omega t - k_x x - k_y y - k_z z + \theta)\mathbf{e}_z$$

Comparing (4) and (3) we obtain:

$$E_{0x} = (\gamma\cos\alpha\cos\beta + \sin\alpha\sin\beta)E_{0x}^{'} - (u\cos\beta)B_{0z}^{'}$$
$$\Omega = \Omega' b_{44} - k_y^{'} b_{24}$$
$$k_x = k_y^{'} b_{21} - \Omega' b_{41} \quad (5)$$
$$k_y = k_y^{'} b_{22} - \Omega' b_{42}$$
$$k_z = 0$$
$$\theta = \theta'$$

$$c = \frac{\Omega}{\sqrt{k_x^2 + k_y^2}} = \frac{\Omega' b_{44} - k'_y\, b_{44}}{\sqrt{\left(k'_y\, b_{21} - \Omega' b_{41}\right)^2 + \left(k'_y\, b_{22} - \Omega' b_{42}\right)}}$$
$$\frac{\Omega'^2}{k'^2}\left(c^2 b_{41}^2 + c^2 b_{42}^2 - b_{44}^2\right) - 2\frac{\Omega'}{k'}\left(c^2 b_{21} b_{41} + c^2 b_{22} b_{42} - b_{24} b_{42}\right)$$
$$+\left(c^2 b_{21}^2 + c^2 b_{22}^2 - b_{24}^2\right) = 0$$
$$c^2\left(b_{41}^2 + b_{42}^2\right) - b_{44}^2 = c^2\left(\left(\frac{u\gamma\sin\beta}{c^2}\right)^2 + \left(\frac{u\gamma\cos\beta}{c^2}\right)^2\right) - \gamma^2$$
$$= \frac{u^2\gamma^2}{c^2} - \gamma^2 = -1$$
$$c^2\left(b_{21} b_{41} + b_{22} b_{42}\right) - b_{24} b_{42} = -u\gamma\sin\beta(\sin\alpha\cos\beta -$$
$$-\gamma\cos\alpha\sin\beta) + u\gamma\cos\beta(\sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta) -$$
$$-u\gamma^2\cos\alpha = 0$$
$$c^2\left(b_{21}^2 + b_{22}^2\right) - b_{24}^2 = c^2\left[(\sin\alpha\cos\beta - \gamma\cos\alpha\sin\beta)^2 +\right.$$
$$\left. + (\sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta)^2\right] - (u\gamma\cos\alpha)^2$$
$$= c^2\left(\sin^2\alpha + \gamma^2\cos^2\alpha\right) - u^2\gamma^2\cos^2\alpha = c^2$$
$$-\frac{\Omega'^2}{k'^2} + c^2 = 0$$
$$\frac{\Omega'}{k'} = \pm c \quad (6)$$

We can now calculate the phase light speed in the rotating frame:

$$v_p^{'} = \frac{\Omega'}{k'} = \frac{\Omega}{k} = \pm c \quad (7)$$

So, the light speed in the rotating frame equals the light speed in the inertial frame, c.

We can now proceed to calculating the amplitude and the phase transformation between the inertial and the rotating frame:

$$\theta = \theta' \quad (8)$$

The general equation of the Doppler effect is:

$$\Omega = \Omega' b_{44} - k_y^{'} b_{24} = \Omega'(b_{44} - \frac{b_{24}}{c}) = \Omega'(\gamma - \frac{u\gamma\cos\alpha}{c}) = \gamma(1 - \frac{u\cos\alpha}{c})\Omega'$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$
$$u = r\omega$$
$$\alpha = \omega\gamma\tau = \omega t \quad (9)$$

This means that the frequency at the Mossbauer receiver is a periodic function of time:

$$\Omega(t) = \gamma(1 - \frac{u\cos\omega t}{c})\Omega_0^{'} \quad (10)$$

Since the receiver integrates all the frequencies received during an integer number of full revolutions of the transmitter, the resultant frequency is:

$$\Omega = \gamma\Omega_0^{'}\frac{\int_0^{k\frac{2\pi}{\omega}}(1 - \frac{u\cos\omega t}{c})dt}{k\frac{2\pi}{\omega}} = \gamma\Omega_0^{'} \quad (11)$$

The above confirms that the Mossbauer effect measures the purest form of transverse Doppler effect. On an interesting note:

$$\gamma\Omega_0^{'} = \frac{\Omega_0^{'}}{\sqrt{1 - \frac{u^2}{c^2}}} = \Omega_0^{'}[1 + \frac{1}{2}\frac{u^2}{c^2} + R_n(\frac{u^2}{c^2})] \quad (12)$$

where $R_n(\frac{u^2}{c^2})$ is the Lagrange remainder of the Taylor series:

$$R_n(\frac{u^2}{c^2}) = \frac{3}{8}(\frac{u^2}{c^2})^2$$
$$\gamma\Omega_0^{'} = \frac{\Omega_0^{'}}{\sqrt{1 - \frac{u^2}{c^2}}} \approx \Omega_0^{'}[1 + \frac{1}{2}\frac{u^2}{c^2}(1 + \frac{3}{4}(\frac{u^2}{c^2}))] \quad (13)$$

The deviation from Kundig's approximation [5]:

$$\gamma\Omega_0^{'} = \frac{\Omega_0^{'}}{\sqrt{1 - \frac{u^2}{c^2}}} \approx \Omega_0^{'}(1 + \frac{1}{2}\frac{u^2}{c^2}) \quad (14)$$

is extremely small given that $u \approx 54 m/s$.

If the integration is not done over an integer number of full revolutions of the transmitter, the sinusoidal term can introduce some small errors as well, for example:

$$\gamma\frac{\int_0^{kT+T/4}(1 - \frac{u\cos\omega t}{c})dt}{kT} = \gamma(1 + \frac{T/4}{kT} + \frac{\frac{u}{c}\sin x\,|_0^{\pi/2}}{kT}) = \gamma[1 + \frac{1}{k}(\frac{1}{4} + \frac{\frac{u}{c}}{2\pi})] \quad (15)$$

Only when the integration occurs over precisely an integer number of revolutions, will the departure from γ be null. But it is very difficult to ensure that the integration occurred over precisely an integer number of revolutions. We can see that the error will vary with the total number of revolutions, k and with the tangential speed of the emitter. It is also easy to see that the error can be minimized by extending the total number of revolutions towards a very large number. The error in this case can be much larger, as it can be gleaned from (15). This type of error motivated Kolmetskii [10,11] to develop a whole new theory of gravitation and ignited a controversy with Corda who proved that such a theory is not only not needed but also incorrect [12].

Lastly, from [8,9] we obtain the equations of aberration:

$$k_x = k'_y b_{21} - \Omega' b_{41} = k' b_{21} - \Omega' b_{41} = k'\left(b_{21} - b_{41} c\right)$$
$$= k'\left(\sin\alpha\cos\beta - \gamma\cos\alpha\sin\beta - \frac{u\gamma\sin\beta}{c}\right)$$
$$k_y = k'_y b_{22} - \Omega' b_{42} = k' b_{22} - \Omega' b_{42} = k'\left(b_{22} - b_{42} c\right)$$
$$= k'\left(\sin\alpha\sin\beta - \gamma\cos\alpha\cos\beta - \frac{u\gamma\cos\beta}{c}\right) \quad (16)$$
$$k_z = 0$$
$$\alpha = \omega\gamma\tau = \omega t$$
$$\beta = \omega\gamma^2\tau = \gamma\omega t$$
$$u = r\omega$$

The above provides us with an interesting finding, the light ray that follows a straight line (radial) path in the rotating frame of the rotor, will follow a curve in the frame of the lab. The x and y directions of the curve are given by:

$$k' = k_y^{'} = 1$$
$$k_x = \sin\alpha\cos\beta - \gamma\cos\alpha\sin\beta - \frac{u\gamma\sin\beta}{c} \quad (17)$$
$$k_y = \sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta + \frac{u\gamma\cos\beta}{c}$$

The light ray directions are time varying. It is also interesting to notice that the reverse is also true, the light ray that follows a straight line (radial) path in lab, will follow a curve in the frame commoving with the rotor:

$$k_x^{'} = k(\sin\alpha\cos\beta - \gamma\cos\alpha\sin\beta - \frac{u\gamma\sin\beta}{c})$$
$$k_y^{'} = k(\sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta + \frac{u\gamma\cos\beta}{c}) \quad (18)$$
$$k_z^{'} = 0$$

The behavior is similar to the Coriolis effect.

3. CONCLUSION

We constructed the rigorous theory of the Mossbauer rotor experiment based on the formalism of special relativity applied to rotating frames. The results show that the experiment is a pure expression of the transverse Doppler effect. In the process of deriving the formalism and we exposed a potential source of very small systematic errors.

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