A Modified Michelson Interferometer and an Application on Microscopic Imaging

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Abstract- We propose a digital cascaded two-beam interference considered as a cosine function of higher order n greater than one (n > 1). We investigate the fringe shift of the microscopic object w.r.t the cascaded two-beam interference of higher orders of n = 1, 2, 3, ..., 25. The fringe sharpness of the cascaded higher order two-beam is compared with the ordinary two beam interference. A modulated multiple beam interference is compared with the improved two beam model for n= 25. The refractive index of the microscopic optical fiber is extracted from the fringe shift of the improved cascaded two beam interference. Mat- Lab code is used to fabricate the investigated interference modulated images.

Key words: Modified Michelson Interferometer, Refractive index of fibers, Digital interference fringes.

1. INTRODUCTION

The two beam interference originated from either division of amplitude or wave front giving intensity distribution proportional to $\cos^2\theta$, like in Michelson or other two beam interferometer. The sharpness is improved using multiple beam interference like in Fabry - Perot interferometer giving the well known Airy distribution formula proportional to $1/[1+F \sin^2\theta]$, where F= $4R/(1-R^2)$. A huge amount of applications based on measuring the fringe shift in optical and synthetic fibers in order to get refractive index information [1-11, 14] are outlined. Recently, digital two and multiple beam interference are applied on Corona virus image and other medical images [12, 13] in order to extract refractive index which is related to the fringe shift.

Now, in the present work we suggest cascaded two beam interference arrangement giving intensity of higher orders of $\cos^2\theta$ in the form of $\cos^{2n}\theta$, where n represents the number of feedback passes to the interferometer. This will sharpen the fringes giving sharper fringes than the corresponding sharpness in case of ordinary two beam interference. The cascaded two beam interferometer is presented followed by theoretical analysis. The results and discussion are given followed by a conclusion.

2. THE CASCADED TWO BEAM INTERFEROMETER BASED ON DIVISION OF AMPLITUDE

As shown in the Fig. 1, L: Laser beam of monochromatic wavelength at $\lambda = 633$ nm. Obj: low numerical objective lens of NA = 0.5. P: pinhole placed in the focal plane of the objective lens of diameter allowing to pass only the central band of the diffraction pattern and L1 is placed at a distance from the pinhole = focal length of L1. Parallel rays of uniform intensity are originated from the Gaussian beam. M1 and M2 are reflecting mirrors corresponding to the original Michelson interferometer where the object is placed in the path of the beam reflected by the mirror M1. In absence of these feedback loops only cos² function is captured by the detector in the observation plane.

Now, the presence of feedback mirrors M3 and M4 and the beam splitters B.S.2 – B.S.6 allows attack the interferometer three times. Hence, three loops corresponding to the three feedback passes allows incidence on the interferometer giving (cos²) ³ = cos⁶ function in addition to the original cos² function of the interferometer. Consequently, cos⁶ function is fabricated in the observation imaging plane. Hence, more feedback passes of the beam give further improvement of sharpness as compared with the ordinary two beam cos² function.

3. THEORETICAL ANALYSIS

The coherent multiplication of two beam interference is governed by the feedback of the coherent laser beam that incident on the Michelson arrangement. Hence, for the number of feedback passes (N) on the interferometer gives intensity distribution in the form:

$$I_{feedback}(x, y; N) = I_0 \cos^{2(N+1)}(\delta)$$  \hspace{1cm} (1)

The ordinary two beam interference has intensity distribution extracted from equation (1), where N = 0(no feedback) as follows:

$$I_{ordinary}(x, y; 0) = I_0 \cos^2(\delta)$$  \hspace{1cm} (2)

For a single feedback N= 1

$$I_{feedback}(x, y; N = 1) = I_0 \cos^4(\delta)$$  \hspace{1cm} (3)

For two feedback light passes, N= 2

$$I_{feedback}(x, y; N = 2) = I_0 \cos^8(\delta)$$  \hspace{1cm} (4)

It is known that each feedback pass adds $\cos^2\delta$ to the original $\cos^2\delta$.

The fringe sharpness is conveniently measured by their full intensity width at half maximum (FWHM) [15]. In the case of ordinary two beam interference, the fringe sharpness is computed as:

$$\frac{I}{I_0} = \frac{1}{2} = \cos^2(\delta_w)$$  \hspace{1cm} (5)

In this case, the FWHM represented by $\delta_w$ is computed as follows:

$$\delta_w = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$  \hspace{1cm} (6)

While in case of multiple feedback of number of passes N, the FWHM is computed as follows:

$$\delta_w = \cos^{-1}\left(\frac{1}{\sqrt{2(N+1)}}\right)$$  \hspace{1cm} (7)
If we consider multiple reflections of the reference beam over the object beam multiple beam interference is obtained modulated by the object information. In this case, the intensity distribution of multiple beam interference is well known as the Airy pattern and represented as follows:

\[ I(x, y, z) = I_0 \frac{1}{1 + F \sin^2 \left( \frac{\delta}{2} \right)} \ \Omega \text{O.P.D.} \]  

(8)

Where \( \delta \) is the phase difference between the object beam and multiple reference beam and O.P.D. is the optical path difference.

The equation (8) is written in discrete form as follows:

\[ I(x, y, z) = I_0 \sum_1^N \sum_1^M \left\{ \frac{1}{1 + F \sin^2 \left( \frac{\delta}{2} \right)} \right\} \]

(9)

The fringe width in case of multiple beam interference is given in [16] as follows:

\[ \delta_w = \frac{4}{\sqrt{F}} \quad F = 4R/(1 - R)^2 \]  

(10)

R is the reflectivity of the plate surface. The fringe sharpness is improved for higher reflectivity.

4. COMPUTATION OF THE REFRACTIVE INDEX OF MICROSCOPIC IMAGES USING THE MODIFIED MICHELSON INTERFEROMETER

The refractive index of any microscopic image in case of modified Michelson interferometer is computed as follows:

Since the phase of the wave cumulates traveling a distance \( L \) in a medium is

\[ \delta(x, y) = \int k \, dl = \int \frac{\mu(x, y, z) \omega}{c} \, dl \]  

(11)

Then, the same wave that propagates over two equivalent paths \( L \) in the microscopic object and in vacuum gives the phase difference as follows (Fig. 2):

\[ \Delta \delta(x, y ; z) = \frac{2\pi}{\lambda} \int \left[ \mu(x, y, z) - 1 \right] \, dl \]  

(12)

Where \( k = \omega/c = 2\pi/\lambda \) is the propagation wave number in a medium of refractive index \( \mu \) and \( k_0 \) is the propagation constant in vacuum.

Finally, from equation (12), the refractive index of the microscopic image is obtained as follows [13, 16, and 17]:

\[ \mu(x, y)_{\text{const}, x} = 1 + a(x, y) \frac{\delta_z}{\Delta z} \]  

(13)

The fringe shift is \( \delta_z \) with respect to inter-fringe spacing \( \Delta z \) at constant \( x \), the fringes are assumed located in the \( x \)- \( y \) plane and \( z \) is the axis normal to the fringe system which represents the height depth and \( a(x, y) \) represents the amplitude of the image. In equation (13),

\[ h(x, y, z) = a(x, y) \delta_z \]  

(14)

Fig. 1: A cascaded higher order two beam interference arrangement of multiples of \( \cos^2 \) function.

Fig. 2: The propagation of light in the object of refractive index \( \mu \) compared with air.

Fig. 3: A segment from an optical fiber and the corresponding modulated two beam interference. Both of the images are of dimensions \( 256 \times 256 \) pixels.

Fig. 4: Modulated two beam interference. The image on the left represent the ordinary \( \cos^2 \delta \) where \( N=0 \), while the image on the right has \( \cos^2 \delta \) where \( N=4 \).
5. RESULTS AND DISCUSSION

A segment from optical fiber and the corresponding modulated two-beam interference are shown as in the Fig. 3. Both of the images are of dimensions 256×256 pixels. Modulated two beam feedback interference is shown as in the Fig. 5: Modulated two beam interference. The image on the left represents \( \cos^{20}\delta \) where \( N=9 \), while the image on the right has \( \cos^{50}\delta \) where \( N=24 \).

Fig. 6: Modulated interference. The image on the left represents two beam \( \cos^{50}\delta \) interference for \( N=24 \), while the image on the right has ordinary multiple beam interference. In all modified two beam images, the intensity in the following form: \( I = I_0 \cos^{2(N+1)}\delta \).

Fig. 7: The image represents two beam in the form \( I = I_0 \cos^{50}\delta \) for \( N=24 \) used in the computation of refractive index of the fiber segment. The discontinuous line is adjusted over the fringe shift to give accurate values. The image has matrix dimensions of 512×512 pixels. The computations applied on six fringes only shown in the image.

Fig. 4. The image on the right has \( \cos^{10}\delta \) originated from 4 feedback passes (\( N=4 \)) compared with the ordinary \( \cos^{5}\delta \). It is shown by the naked eye that the multiple feedback passes on two beam interference improves sharpness of the fringes as compared with the ordinary two beam interference. Further improvement of fringe sharpness is attained by increasing the number of feedback passes as shown in the Fig. 5, where \( N = 9 \) and \( N = 24 \). In addition, the two beam fringes with high feedback passes \( N=24 \) is compared with the ordinary multiple beam interference as shown in the Fig. 6. The fringe sharpness in case of multiple feedback passes in two beam arrangement is computed using equation (7), while the ordinary fringe sharpness is computed from equation (6). Some results of the fringe sharpness computed from FWHM using equation (7) are constructed as in the table (1). It is shown that sharpness in case of feedback passes in the modified Michelson arrangement is much better than the corresponding sharpness in the ordinary two beam interference. The sharpness improvement is dependent on the number of feedback passes \( N \). For example, the sharpness is reduced from 50 for ordinary two-beam to 2.38 in case of multiple feedback passes where \( N=9 \) for \( I = I_0 \cos^{2(N+1)}\delta = I_0 \cos^{20}\delta \).

Table (1): Fringe sharpness in case of feedback passes compared with ordinary two beam interference in a Michelson arrangement.

<table>
<thead>
<tr>
<th>Two beam intensity</th>
<th>N</th>
<th>Fringe sharpness</th>
</tr>
</thead>
<tbody>
<tr>
<td>with feedback</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I = I_0 \cos^{5}\delta )</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>( I = I_0 \cos^{7}\delta )</td>
<td>1</td>
<td>36.39</td>
</tr>
<tr>
<td>( I = I_0 \cos^{9}\delta )</td>
<td>2</td>
<td>26.12</td>
</tr>
<tr>
<td>( I = I_0 \cos^{11}\delta )</td>
<td>3</td>
<td>18.6</td>
</tr>
<tr>
<td>( I = I_0 \cos^{13}\delta )</td>
<td>4</td>
<td>13.19</td>
</tr>
<tr>
<td>( I = I_0 \cos^{15}\delta )</td>
<td>5</td>
<td>9.36</td>
</tr>
<tr>
<td>( I = I_0 \cos^{17}\delta )</td>
<td>6</td>
<td>6.62</td>
</tr>
<tr>
<td>( I = I_0 \cos^{19}\delta )</td>
<td>7</td>
<td>4.68</td>
</tr>
<tr>
<td>( I = I_0 \cos^{21}\delta )</td>
<td>8</td>
<td>3.37</td>
</tr>
<tr>
<td>( I = I_0 \cos^{23}\delta )</td>
<td>9</td>
<td>2.38</td>
</tr>
</tbody>
</table>
The refractive index of the fiber is computed from equation (13), where \( a(x, y) = 1 \) and the average inter-fringe spacing is computed from the image shown in the Fig. 7 to give \( \Delta Z = 78 \) pixels. The fringe shift \( \delta Z = Z - Z_{\text{image}} \) is computed for six fringes taken from the same Fig. 7. Finally, the average refractive index from the table (2) is computed to give \( \langle \mu \rangle = \left(1/M\right) \sum_{m=1}^{M} \mu(Z) = 1.5976 \).

Table (2): The refractive index values as a function of the Z coordinate at certain line adjusted over the fringe shift at (297,280) pixels. \( \Delta Z = 78 \) pixels represent the average inter-fringe spacing. The shift of the modulated fringes is towards the right.

<table>
<thead>
<tr>
<th>Z</th>
<th>Z_{\text{image}}</th>
<th>\delta Z = Z - Z_{\text{image}}</th>
<th>\mu(Z) = 1 + \delta Z/\Delta Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>90</td>
<td>50</td>
<td>1.6757</td>
</tr>
<tr>
<td>117</td>
<td>172</td>
<td>55</td>
<td>1.7051</td>
</tr>
<tr>
<td>197</td>
<td>248</td>
<td>51</td>
<td>1.6538</td>
</tr>
<tr>
<td>274</td>
<td>320</td>
<td>46</td>
<td>1.5897</td>
</tr>
<tr>
<td>344</td>
<td>384</td>
<td>40</td>
<td>1.5128</td>
</tr>
<tr>
<td>420</td>
<td>455</td>
<td>35</td>
<td>1.4487</td>
</tr>
</tbody>
</table>

In the Fig. 8, fringe shift of fiber in multiple beam interference where 14 fringes are shown in the image of dimensions 512x512 pixels using the Airy distribution formula represented by equation (9).

Another application on metal inspection using multiple beam interference is shown. The image of water droplets agglomerated on the aluminum surface of dimensions 512x512 pixels is shown as in the Fig. 9. The corresponding multiple beam interference is plotted as in the Fig. 1 where the random irregular fringe shifts corresponding to the agglomerated droplets are shown. In the upper left, a segment from the whole image at \( (i, j) = (50 - 150 \) pixels, \( j = 200 - 300 \) pixels) where magnified droplet from the aluminum surface is shown as in the Fig. 11 (a). The other images, shown in the Figs 11 (b-d), are corresponding to 3, 6, and 9 fringes of modulated multiple beam interference. It is outlined that all interference images are digitally constructed using Mat-Lab code used in [17].

6. CONCLUSION

The feedback coherent light is suggested in two beam interference of Michelson arrangement. The intensity distribution in case of feedback of two beam interference is written and the corresponding fringe sharpness is computed. A comparison with the ordinary two beam fringe sharpness is discussed. It is shown that the fringe sharpness in case of feedback light passes is better than the corresponding fringe sharpness in case of two beam interference. In addition, the fringe sharpness has further improvement with increasing the number of feedback passes. Finally, the refractive index of microscopic images has accurate results as compared with the ordinary two beam interference. Consequently, the final goal of this work based on three objectives is realized. The 1\(^{st}\) is the improvement of fringe sharpness in case of feedback of two beam interference. The 2\(^{nd}\) is the accurate measurement of the fringe shift computed from the considered sharp fringes. The 3\(^{rd}\) is the extraction of refractive index of the fiber specimen from the fringe shift results.

REFERENCES


