In this paper, a theoretical model for analysis of the group delay spectrum of exponential-linear tapered fiber Bragg gratings (TFBGs) under Applied Strain. For this reason, a numerical model based on solving the coupled mode theory that simulates the spectral response of this structure, in order to optimize the dispersion and dispersion slope compensation performance of (TFBGs). We are the first to find that exponential tapered fiber Bragg gratings can be used in dispersion slope compensation. These characteristics of the Bragg grating make it can be utilized efficiently in high bit rates DWDM.

**Key Words:** Bragg grating, Silica optical fiber, Tapered fiber, Reflection, Group delay.

1. INTRODUCTION

With the rapid increase of information industry in the world, the need of high speed and big capacity communication networks become more and more insistent, but the phenomenon of chromatic dispersion is one of the confining factors in long-haul high-speed transmission links. Bragg grating technologies proved a highly effective solution to compensate the chromatic dispersion of high speed optical fiber communication systems [1].

Importance in fiber Bragg gratings (FBGs) has grown increasingly in recent years due to their ease of manufacture and various applications in the field of optical fiber technology. Specifically, they can be efficiently used for dispersion compensation in high-speed long-haul optical communication systems [1-3], short-pulse generation and restoration [4-5]. Besides, FBGs can be used for the implementation of high-quality fiber laser cavities of various geometries [6-7] and semiconductor diode stabilization [8]. Also, FBGs are spectral filters based on the principle of Bragg reflection [9-2]. FBG, first demonstrated by Hill et al. [10], is developed by inscribing periodic refractive index modulation into the core of optical fiber using intense ultraviolet (UV) source through interferometry, point-by-point or phase mask technique [11]. The variation of the refractive index gives rise to a photonic band gap inside their spectrum where linear waves cannot propagate [12].

In long-distance optical communication systems, fiber group velocity dispersion (about 17ps/nm.km for standard fibers) degrades system performance by limiting either the maximum bit rate or the distance length (less than 60 km for standard NRZ format at 10 Gbps) [13]. For 10-Gbps transmission system, the dispersion slope caused high order group delay is negligible [14]; but for high speed systems operating at 40 Gbps and beyond the dispersion slope has to be compensated. In recent years, there has been increasing interest in the study of linearly chirped fiber Bragg gratings with different apodization profiles in order to be used as compensator devices in high bit rate systems. Indeed, because fiber Bragg gratings are easy to make, inexpensive, low-insertion loss, compact, compatible with all fiber communication systems.

Crucial importance is to study the dispersion and dispersion slope characteristics of tapered FBGs under stress. In ref. [14-15], the authors have analyzed the dispersion characteristics of linearly tapered FBG. It was demonstrated, theoretically and experimentally, that linearly tapered FBGs display nonlinear group delay under strain, which means that the linearly tapered FBGs can be used in dispersion slope compensation.

In this work, utilizing the same technique of calculus as in ref. [14-15], we discuss, theoretically and numerically, the dispersion characteristics of tapered FBGs having exponential-linear taper profile. Our results are compared to those reported in ref. [14-16].

2. THEORY OF TFBGs UNDER STRAIN

The structure of tapered fiber Bragg grating is illustrated in Fig.1. The function for the Exponential-linear taper profile is given by [6].

\[
R(z) = R_0 \left[ e^{z/z_0} - \frac{z}{z_0} e^{-1} \right] 
\]  

\( R_0 \) represents the initial fiber radius, \( z_0 \) the initial fiber height and \( z \) the propagation length. In other words, the fiber radius changes exponentially from \( R(z) \) to \( R(z_0) \), where \( R(z) \) is given by [6].

Fig. 1: Illustration of tapered FBG (a) and tapered FBG after applied strain (b).
where $R_0$ is the original radius of the fiber and $z_0$ is the point along the $z$ axis where the radius of the tapered fiber may come to zero.

The cross-section area of the grating at position $z$ is defined by

$$A(z) = \pi R(z)^2 = \pi R_0^2 \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2$$  \hspace{1cm} (2)

While tension $F$ is applied to the grating, the axial strain $\varepsilon(z)$ can be expressed as [7].

$$\varepsilon(z) = \frac{F}{E A(z)} = \frac{F}{E \pi R_0^2 \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2} = \varepsilon(0) / \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2$$ \hspace{1cm} (3)

where $E$ is Young’s modulus of the fiber, and $\varepsilon(0) = F / E \pi R_0^2$ is the strain at $z = 0$.

The change of the grating period along $z$ axis may be expressed as

$$\Delta \Lambda(z) = \varepsilon(z) \Lambda_0 = \varepsilon(0) \Lambda_0 / \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2$$

$$= \Delta \Lambda(0) / \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2$$ \hspace{1cm} (4)

where $\Lambda_0$ is the initial grating period at position $z=0$. Hence, the period under tension along the $z$ axis changes to

$$\Lambda(z) = \Lambda_0 + \Delta \Lambda(z) = \Lambda_0 + \Delta \Lambda(0) / \left( e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} \right)^2$$ \hspace{1cm} (5)

When the taper slope is very small, that means $z \ll z_0$, the relationship between $z$ and $z'$ can be expressed as

$$z' = z + \frac{\Delta \Lambda(0)}{\Lambda_0} z = z \left( 1 + \frac{\Delta \Lambda(0)}{\Lambda_0} \right)$$ \hspace{1cm} (6)

To analyse the group-delay characteristics, we can rewrite the Eq. (5) in the form

$$e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1} = \left( \frac{\Delta \Lambda(0)}{\Lambda(z) - \Lambda_0} \right)^{1/2}$$ \hspace{1cm} (7)

we expand $e^{\frac{z}{z_0}} - \frac{z}{z_0} e^{-1}$ in Taylor series around $z = 0$, we obtain

$$e^{\frac{z}{z_0}} = 1 - \frac{z}{z_0} + \frac{z^2}{2z_0^2} + ...$$

by neglecting the term of order two in Eq. (8), we can rewrite the Eq. (7) in the form

$$z = \frac{z_0}{1 + e^{-1}} \left( 1 - \frac{\Lambda(z) - \Lambda_0}{\Delta \Lambda(0)} \right)^{-1/2}$$ \hspace{1cm} (9)

Expanding equation (9) in Taylor series at $z = 0$, we obtain

$$z = \frac{z_0}{1 + e^{-1}} \left( 1 - \frac{1}{2 \Delta \Lambda(0)} (\Lambda(z) - \Lambda(0)) \right) - \frac{3}{8 \Delta \Lambda(0)^2} (\Lambda(z) - \Lambda(0))^2$$ \hspace{1cm} (10)

using (6) and (10), we obtain

$$z' = \frac{z_0}{1 + e^{-1}} \left( 1 - \frac{1}{2 \Delta \Lambda(0)} (\Lambda(z) - \Lambda(0)) \right) - \frac{3}{8 \Delta \Lambda(0)^2} (\Lambda(z) - \Lambda(0))^2 \left( 1 + \frac{\Delta \Lambda(0)}{\Lambda_0} \right)$$ \hspace{1cm} (11)

Considering the refractive index changes with the applied strain because of the photo-elastic effect, the change of Bragg wavelength at original point is given by [8].

$$\Delta \lambda_g(z) = \varepsilon(z)(1 - \chi) \lambda_0 = \varepsilon(z)(1 - \chi) \cdot 2n_{eff} \Lambda_0 \hspace{1cm} (12)$$

$$= \Delta \Lambda(z)(1 - \chi) 2n_{eff}$$

where $\chi$ is the photo-elastic effect and $E$ is the Young’s modulus, which describes the fiber lengthening effect ($\chi = 0.22$ for silica).

According to Eqs. (11) and (12), we can express $z'$ as a function of the Bragg wavelength variation

$$z' = \frac{z_0}{1 + e^{-1}} \left( 1 - \frac{1}{2 \Delta \Lambda(0)} \left( \frac{\Delta \lambda_g(z)}{(1 - \chi) 2n_{eff}} \right) \right) - \frac{3}{8 \Delta \Lambda(0)^2} \left( \frac{\Delta \lambda_g(z)}{(1 - \chi) 2n_{eff}} \right)^2 \left( 1 + \frac{\Delta \Lambda(0)}{\Lambda_0} \right) \hspace{1cm} (13)$$

The group delay experienced by a signal reflected from a particular position $z'$ of the grating is given by $t = 2z' n_{eff} / C$, where $C$ is the speed of light in vacuum.

$$t = \frac{2n_{eff} z_0}{(1 + e^{-1}) c} \left( 1 + \frac{\Delta \Lambda(0)}{\Lambda_0} \right) \left( \frac{1}{2 \Delta \Lambda(0)} \left( \frac{\Delta \lambda_g(z)}{(1 - \chi) 2n_{eff}} \right) \right) - \frac{3}{8 \Delta \Lambda(0)^2} \left( \frac{\Delta \lambda_g(z)}{(1 - \chi) 2n_{eff}} \right)^2 \hspace{1cm} (14)$$
3. RESULT AND DISCUSSION
The different parameters used in the simulation are as follows.

For the uniform fiber Bragg grating parameters are approximately $L_0 = 80 \text{mm}$ in length without applied tension. The original grating period is $\Lambda_0 = 531.8 \text{nm}$. The refractive index at the beginning of fiber core $n_{eff}(0) = 1.452$, the diameter varying from $125 \mu\text{m}$ to $107 \mu\text{m}$. The applied strain to the tapered grating is $0.0028 \text{N}$.

We have studied the group delay ripples (GDR) and the dispersion slope induced by taper profile for no apodized FBG in Fig 2. a. and tapered FBG with Gaussian apodization (the Gaussian parameter has been taken as $G = 15$) in Fig 2. b. It can be seen from Eq. (14) that the tapered FBG grating under strain has a nonlinear group delay characteristics (dispersion slope). Moreover, the linear coefficient (i.e. coefficient of the first term of Eq. (14)) and the quadratic coefficient (i.e. coefficient of the second term of Eq. (14)) of the group delay characteristics are positive and negative sign respectively. These coefficients can be determined by means of the different profiles used in the simulation.

We can clearly see that in the case of exponential-linear profile, the taper radius decreases faster along the grating axis (see Fig. 1), consequently the period of grating increase more rapidly.

Fig. 2 shows the variation of the time delay response as function of the wavelength for exponential-linear taper Bragg grating under strain profile. We observe that the group delay ripples is about $\pm 14 \text{ps}$ peak to peak because the tapered FBG is not apodized. Apodization technique is, generally, used to get optimized spectra with side lobes suppression. Consequently, the GDR for the tapered FBG with Gaussian apodization is largely reduced, it is about $\pm 3.06 \text{ps}$. The linear and quadratic coefficients of the group delay response can be theoretically estimated as $143.6 \text{ps/ nm}$ and $-31.7 \text{ps/ nm}^2$, respectively. These characteristics of the grating, which typically exhibits a dispersion slope of $0.06 \text{ps/ nm}^2 / \text{km}$, can be used to compensate for chromatic dispersion up to $528.3 \text{km}$ in long link of single-mode fiber. Then, the longest compensated transmission distance is less than the distance reported in Ref [14]. Indeed, the absolute value of the dispersion slope calculated from equation Eq. (14) is less than the value calculated by Zhang et al. [14], as shown in Table 1. However, devices based on this kind of tapered fiber Bragg grating are potential to play a key role in high speed optical fiber communication systems which need dispersion and dispersion slope (high order dispersion) to be compensated at the same time [16].

Table 1: Variation of GDR and dispersion slope for exponential-linear tapered fiber Bragg grating under strain.

<table>
<thead>
<tr>
<th>Taper profile</th>
<th>Dispersion slope(ps/nm²)</th>
<th>GDR (ps)</th>
<th>Compensating Fiber (Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp-linear</td>
<td>-31.7</td>
<td>3.06</td>
<td>528</td>
</tr>
<tr>
<td>Linear [14]</td>
<td>-43.3</td>
<td>6</td>
<td>721</td>
</tr>
</tbody>
</table>

4. CONCLUSION
In summary, linearly tapered fiber Bragg grating under strain is more effective than exponential-linear tapered grating for dispersion slope compensation. Our work shows that the dispersion slope of 528 km of standard fiber at 1550 nm can be perfectly compensated with a grating with an exponential-linear tapered profile against 721 km for linear tapered grating. Finally, the results could be valuable and should be taken into account for the TFBGs design, and some special applications for the fiber communications.

REFERENCES:


