Method and Experimental Setup for Constraining One Way Light Speed Anisotropy

Adrian Sfarti
University of California, 387 Soda Hall, UC Berkeley, California, USA
egas@pacbell.net

(Received 31st August, 2016; Accepted 23rd September, 2016; Published: 25th September, 2016)

Abstract- In the current paper we will set the foundations for the Doppler effect in the framework of the Mansouri-Sexl test theory. The measurements of the relativistic Doppler effect via the Ives-Stilwell experimental setup enable us to generate strong constrains on Lorentz symmetry violations. The subject is of outmost importance in all experiments that constrain the Lorentz violations via Ives-Stilwell type tests executed under the Mansouri-Sexl formalism.

Key Words: Mansouri-Sexl test theory, Relativistic Doppler effect, One way light speed anisotropy constraints.

1. THEORETICAL FOUNDATIONS OF THE MANSOURI-SEXL TEST THEORY

Different test theories differ in their assumptions about the form of the Lorentz transforms. The main test theories of special relativity (SR) are named after their authors, Mansouri and Sexl [1]. The Mansouri-Sexl test theories can also be used to examine potential alternate theories to SR - such alternate theories predict particular values of the parameters of the test theory, which can easily be compared to the values determined by experiments analyzed with the test theory. The existing experiments [3-8] put rather strong experimental constraints on any alternative theory. The Mansouri-Sexl test theory starts by assuming that there is a preferential inertial frame \( \Sigma \) in which the light in a vacuum propagates isotropically with the speed \( c \). All other frames in motion with respect to \( \Sigma \) are considered non-preferential and the light speed is anisotropic. The Mansouri-Sexl theory, alongside with the Standard Model Extension, has become one of the two primary test theories for the special relativity. We start our derivation with the Mansouri-Sexl transforms for the case of frames with aligned axes of coordinates. This is exactly the setup employed by the Ives-Stilwell experiment [2, 4]:

\[
x = b(w)(X - wT)
\]

\[
t = a(w)T + \varepsilon(w)x = (a(w) - b(w)\varepsilon(w))T + b(w)\varepsilon(w)X
\]

where \( w \) is the relative speed between the ion frame \( S \) and \( \Sigma \) and \((x,t)\) are the coordinates in \( S \) and \((X,T)\) represent their correspondents in \( \Sigma \). Let a lab frame \( S' \) move with speed \( w' \) with respect to frame \( \Sigma \). The transforms between lab frame \( S' \) and frame \( \Sigma \) are:

\[
X = \frac{1}{b'(w')} - \frac{w'\varepsilon'(w')}{a'(w')} x' + \frac{w'}{a'(w')} t'
\]

\[
T = \frac{t' - \varepsilon'(w') x'}{a'(w')} = -\frac{\varepsilon'(w')}{a'(w')} x' + \frac{1}{a'(w')} t'
\]

\[
a'(w') = 1 + \alpha w'^2
\]

\[
b'(w') = 1 + \beta w'^2
\]

In the Mansouri-Sexl theory light speed is isotropic (and denoted from her on as \( c_\alpha \)) only in frame \( \Sigma \). In the lab frame light speed is obtained from:

\[
dX^2 - c_\alpha^2 dT^2 = 0
\]

Neglecting the second order terms, the above equation gives [2]:

\[
\frac{c_\alpha}{c_0} = \frac{1}{1 + 2\alpha}
\]

The speed of light propagating in the negative x axis sense \( c(w') = c_\alpha + (1 + 2\alpha) \) will be used in the derivation of the Mansouri-Sexl Doppler effect. It is interesting to notice that for \( \alpha = -0.5 \) we recover the isotropic light speed of special relativity. Any deviation of the parameter \( \alpha \) from -0.5 is interpreted as one way light speed anisotropy.

2. THE COMPLETE MANSOURI-SEXL THEORY OF THE DOPPLER EFFECT

In [1] the authors have developed a Mansouri-Sexl theory of the Ives-Stilwell experiment that permits constraining the “a” parameter only. In the following, we will give more depth to the theory such as to enable constraining both “a” and “b”. We “revise” both the Mansouri-Sexl theory of the Doppler effect and the theory of the Ives-Stilwell experiment in the light of the Mansouri-Sexl theory. This is necessary in order to obtain a coherent foundation of the Ives-Stilwell experiment as judged from the framework of the test theory. This reformulation is also extremely important for experimental purposes since the more parameters that can be constrained from a single experiment, the more comprehensive the experiment. We will derive the equation of the Doppler effect in the Mansouri-Sexl formalism using a more straightforward approach than the one used by Kretzschmar [2]. The plan is as follows:

- derive the Doppler effect between the lab frame \( S' \) and frame \( \Sigma \)
- derive the Doppler effect between the two ion frames \( S_i \) and frame \( \Sigma \)
- use transitivity in order to derive the desired Doppler effect between the lab frame \( S \) and the two ion frames \( S_i \)

We start with the Doppler effect between the lab frame \( S' \) and frame \( \Sigma \). Assume that a light pulse is emitted at \((X_i, T_i) = (0,0)\) in frame \( \Sigma \). In the lab frame \( S' \) this corresponds...
to \((x'_1, t'_1) = (0, 0)\). Now, a second pulse emitted at 
\((X, T) = (0, T)\) in frame \(\Sigma\) in the lab frame \(S'\) this 
corresponds to \((x'_2, t'_2)\) given by solving:

\[
0 = \left( \frac{b'(w')}{w'e'(w')} \right) x'_2 + \frac{w'}{a'(w')} t'_2.
\]

\[
T = \frac{t'_2 - e'(w')x'_2}{a'(w')}.
\]  \(\text{(5)}\)

The above has the solution:

\[
x'_2 = -b'(w')wT
\]

\[t'_2 = (a'(w') - b'(w')w'e'(w'))T\]  \(\text{(6)}\)

The time interval between the two pulses in the lab frame 
\(S'\) is not simply \(t'_2 - t'_1\), because \(x'_2 \neq x'_1\), it is rather:

\[
T' = t'_2 - t'_1 + \frac{|x'_2 - x'_1|}{c.(w')} = T \left( a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')} \right)
\]  \(\text{(7)}\)

So, the ratio of frequencies is:

\[
u' = \frac{T}{T'} = \frac{1}{a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')}}
\]  \(\text{(8)}\)

Expression (8) represents the general form of the Doppler 
effect in the Mansouri-Sexl formalism between the lab frame 
\(S'\) and the frame \(\Sigma\). From (8) we obtain immediately the 
Doppler effect between the two ion frames \(S_1\) and the frame 
\(\Sigma\):

\[
v_{\nu} = \frac{1}{v' a(w_s) - b(w_s)w_e(w_s) + \frac{b(w_s)w_c}{c.(w_s)}}
\]  \(\text{(9)}\)

We need to remember that \(v_\nu = v_{\nu}\) is the resonance 
frequency of the ions in the Ives-Stilwell experiment, so, (9) 
reduces to:

\[
v' = \frac{1}{v' a(w_s) - b(w_s)w_e(w_s) + \frac{b(w_s)w_c}{c.(w_s)}}
\]  \(\text{(10)}\)

From (8) and (10) we can derive the desired formula for 
Doppler effect between the lab frame \(S\) and the two ion 
frames \(S'_1\), eliminating the effect of the frame \(\Sigma\):

\[
v' = \frac{a(w_s) - b(w_s)w_e(w_s) + \frac{b(w_s)w_c}{c.(w_s)}}{v' a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')}}
\]  \(\text{(11)}\)

\[
v' = \frac{a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')}}{v' a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')}}
\]  \(\text{(12)}\)

We obtain the very elegant final expressions for the 
Doppler effect in the Mansouri-Sexl formalism:

\[
u' = \frac{a(w_s) + b(w_s)w_c}{v' a'(w') - b'(w')w'e'(w') + \frac{b'(w')w'}{c.(w')}}
\]  \(\text{(13)}\)

In the Ives-Stilwell experiment the ions move in both 
parallel and anti-parallel directions, so we will use \(\pm V'\) for 
the former and \(-V'\) for the latter. The same applies to the speed 
of ions in frame. There are three types of speeds in the 
Ives-Stilwell experiment under the Mansouri-Sexl formalism:

\[
\begin{array}{|c|c|}
\hline
\text{Symbol} & \text{Description} \\
\hline
\pm V' & \text{Speed of ions in lab frame } S' \text{(parallel and anti-parallel)} \\
\hline
w_s & \text{Speed of ions in frame } \Sigma \text{ (parallel and anti-parallel)} \\
\hline
w' & \text{Relative speed between frames } S' \text{ and } \Sigma \text{ (see section 4)} \\
\hline
\end{array}
\]

The ion speed in frame \(\Sigma\) is not known and we will show 
later on in the paper how to express it in terms of \(\pm V'\) and 
\(w'\). In (13) \(\nu'\) is the frequency of the light emitted by the ion 
approaching the detector, \(\nu'\) is the frequency of the light 
emitted by the ion receding from the detector, as measured in 
the lab frame and \(v_{\nu}\) is the proper frequency (measured in 
the ion frame). It is clear that the Doppler effect is dependent 
on both parameters \(a\) and \(b\). Contrast (2/9) with (5.3) in 
reference [2]: the dependency on both parameters \(a\) and \(b\) is 
evident. A practical application of the new formalism is 
presented in reference [6]. The problem with (2/9) is the 
dependency in the next section.

3. SPEED COMPOSITION IN THE MANSOURI-SEXL 
FORMALISM

The values for both \(w'\) and \(V'\) are known and the value for 
\(w\) is not, so we need to derive a formalism for determining 
them. From \(w'\) and \(V'\) we can determine the ion speed \(w\) in 
\(\Sigma\) as a function of their speeds measured in the lab frame \(S'\)

\[
w = \frac{dx}{dt} \frac{dx' }{dt'} = \left( \frac{1}{b'} - \frac{w'e'}{a'} \right) \frac{dx' }{dt'} + \frac{w'}{a'}
\]

\[
= \frac{1}{b'} - \frac{w'e'}{a'} \left( \pm V' \right) + \frac{w'}{a'}
\]  \(\text{(14)}\)

The result should be independent of the clock 
synchronization convention, therefore we can choose any 
convention we want. For example, by choosing Einstein 
synchronization:
\[ w_0 = \frac{\pm V'}{b'(w')(1-w'^2)} + \frac{w'}{w' b'(w') + a'(w')} \approx \frac{w' b'(w') + a'(w')}{b'(w') + V' a'(w') w'} \] (15)

According to Mansouri [1], the following approximations hold:

\[ a'(w') = 1 + \alpha w' \]
\[ b'(w') = 1 + \beta w' \] (16)

Substituting (16) into (15) gives \( w_0 \) as a function of the variables \( V', w' \), and the parameters \( \alpha, \beta \):

\[ w_0 = \pm V'(1 + \alpha w') + w' (1 + \beta w') \]
\[ \approx (w' \pm V')(1 - \beta w') \approx w' \pm V' \] (17)

Note that \( \beta w' \) is of the same order as \( V' w' \) (and both are much smaller than unity). So we need to retain both terms. Expression (17) allows us to approximate:

\[ w_0 \approx (w' + V') \approx V' \] (18)

The last piece of the puzzle is substituting (17) and (18) into (13):

\[ \frac{V'}{V_0} \approx \frac{a(w')}{1-w'^2} + \frac{b'(w')w'}{c'(w') w_0} \]
\[ \frac{V'}{V_0} = \frac{a(w')}{1-w'^2} + \frac{b'(w')w'}{c'(w') w_0} \] (19)

One measurement for \( \frac{V'}{V_0} \) and one for \( \frac{V'}{V_0} \) at a given ion speed are sufficient for constraining both parameters \( \alpha, \beta \). One objection is that we do not know \( V_0 \). First hand, we can easily get around this issue by measuring \( \frac{V'}{V_0} \) and \( \frac{V'}{V_0} \) and observing that their ratio is:

\[ \frac{V'}{V_0} = \frac{a(w')}{1-w'^2} + \frac{b'(w')w'}{c'(w') w_0} \]
\[ \approx \frac{1 + \alpha w' + V'}{1 + 2(\alpha + 1)(w' + V')} \]
\[ \approx \frac{1 + \beta w' + V'}{1 + 2(\alpha + 1)(w' + V')} \]
\[ \approx \frac{1 + \alpha w' + V'}{1 + 2(\alpha + 1)(w' + V')} \]
\[ \approx \frac{1 + \beta w' + V'}{1 + 2(\alpha + 1)(w' + V')} \] (20)

Two measurements at two different ion speeds \( V' \) are sufficient for constraining parameters \( \alpha, \beta \). It can be seen that the measurements are more sensitive to \( \alpha \) departing form -0.5 than to \( \beta \) departing from its special relativity value of 0.5. Nevertheless, since the ion speed can be a considerable fraction of the light speed, the influence of parameter \( \beta \) is measurable.

The laboratory speed with respect to the “preferential frame” (taken as the frame in which CMBR is isotropic) \( w' = w'(t') \) has contributions [3, 5-8] from the motion of the Sun with respect to frame \( \Sigma \) with a constant velocity \( V_0 = 377 \text{ km/s} \), while Earth’s orbital motion around the Sun is \( V_0 = 30 \text{ km/s} \). For Berkeley (latitude \( 37^\circ 52' 18'' \mathrm{N} \)) \( V_0 = 0.355 \text{ km/s} \). Putting it all together:

\[ w = v_c + v_s \sin(\Omega_s (t-t_s)) \cos \Phi_s + v_c \sin(\Omega_s (t+t_s)) \cos \Phi_s \] (22)

Here \( \Phi_s \approx 8^\circ \) is the angle between the equatorial plane and the velocity of the Sun. \( \Omega_s = 6^\circ \) is the declination between the plane of Earth’s orbit and the velocity of the Sun, 2\( \pi / \Omega_s = 1 \text{ yr}, 2\pi / \Omega_s = 1 \text{ sidereal day}, \ t_s \) and \( t_s' \) are determined by the phase and start date of the measurement, respectively. In order to constrain the parameters \( \alpha, \beta \) we will take a series of measurements at different ion speeds \( V' \) over periods of time long enough such that we could integrate the sinusoidal effects such that they will reflect in (21).

4. CONCLUSION

We have shown that the correct formalism of the Doppler effect presented entirely in the framework of the Mansouri-Sexl theory must show a dependence of not only parameter “a” as previously shown by Kretzschmar but also of parameter “b”. We have presented the corrected formalism for the Mansouri-Sexl test theory as applied to the Ives-Stilwell experiment and we have derived the speed composition formula for the theory.

REFERENCES

